ME 450 Composite Materials and Design

Title:	Composite Laminate Analysis | Report	
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Table of Contents

List of Tables

1 Introduction and Background

Composite materials and composite fabrication techniques are widely used in modern manufacturing. From commercial airliners to bicycles (Figure 1) to automobiles, composites can be found in almost any industry. This widespread popularity is largely due to the favorable material characteristics that composites have to offer—composite structures boast a much higher strength-to-weight ratio than their metal counterparts, and composites are resistant to corrosion, have long fatigue life, and enable complex manufacturing geometry.

(a) (b)

Figure 1. (a) composite racing bicycle (b) commercial airliner made primarily of composite materials

1.1 Definition of Composites

Composites are materials formed by combining two or more materials that have quite different properties. The different materials work together to give the composite unique properties, but unlike alloys or similarly mixed metals – the materials in composites do not dissolve or blend into each other. Most composites are made up of just two materials. One material (the matrix or binder) surrounds and binds together a cluster of fibers or fragments of a much stronger material. In the case of mud bricks, the two roles are taken by the mud and the straw; in concrete, by the cement and the aggregate; in a piece of wood, by the cellulose and the lignin. In fiberglass, the reinforcement is provided by fine threads or fibers of glass, often woven into a sort of cloth, and the matrix is a plastic.

1.2 Difficulties associated with composite analysis

Due to the inherent anisotropies of composite materials, the computations involved in predicting strengths and other properties of composite structures is no easy task. The analysis contained herein explores a technique known as Classical Lamination Theory (CLT) to predict the effective extensional properties of test specimens cut from twelve composite panels. The assumptions that come along with CLT make failure analysis difficult and it is also difficult to know what values to use for material properties. These difficulties will be further discussed throughout the report.

1.3 Objectives

The purpose of this laboratory exercise is to fabricate carbon fiber and fiberglass laminate composites using hand layup techniques with vacuum bagging and then evaluate the strength properties of these composite panels. The report begins with a discussion of the lay-up process for the twelve test panels followed by a discussion of the use of CLT to predict the effective extension properties and the failure load. A comparison of the predicted properties and the measured properties will be made to see how accurate CLT was, along with any other sources of error.

2 Composite Fabrication | Hand Lay-up with Vacuum Bagging

Twelve composite panels were constructed using hand lay-up techniques. The first step of the process was to place a nonporous peel ply (Figure 2) on the surface of the workstation. The nonporous peel ply prevents resin from passing through onto the tabletop. Also, peel plies do not develop strong bonds to epoxy/composites and can therefore be removed following consolidation.

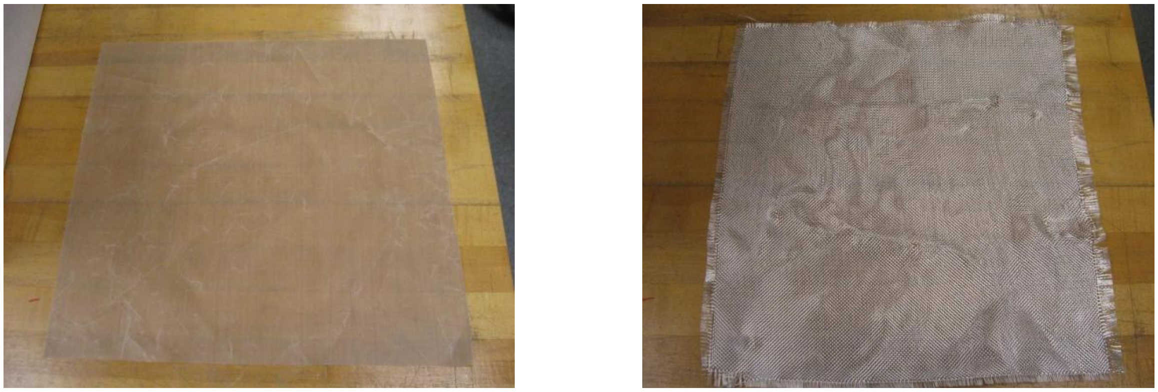

(a) (b)

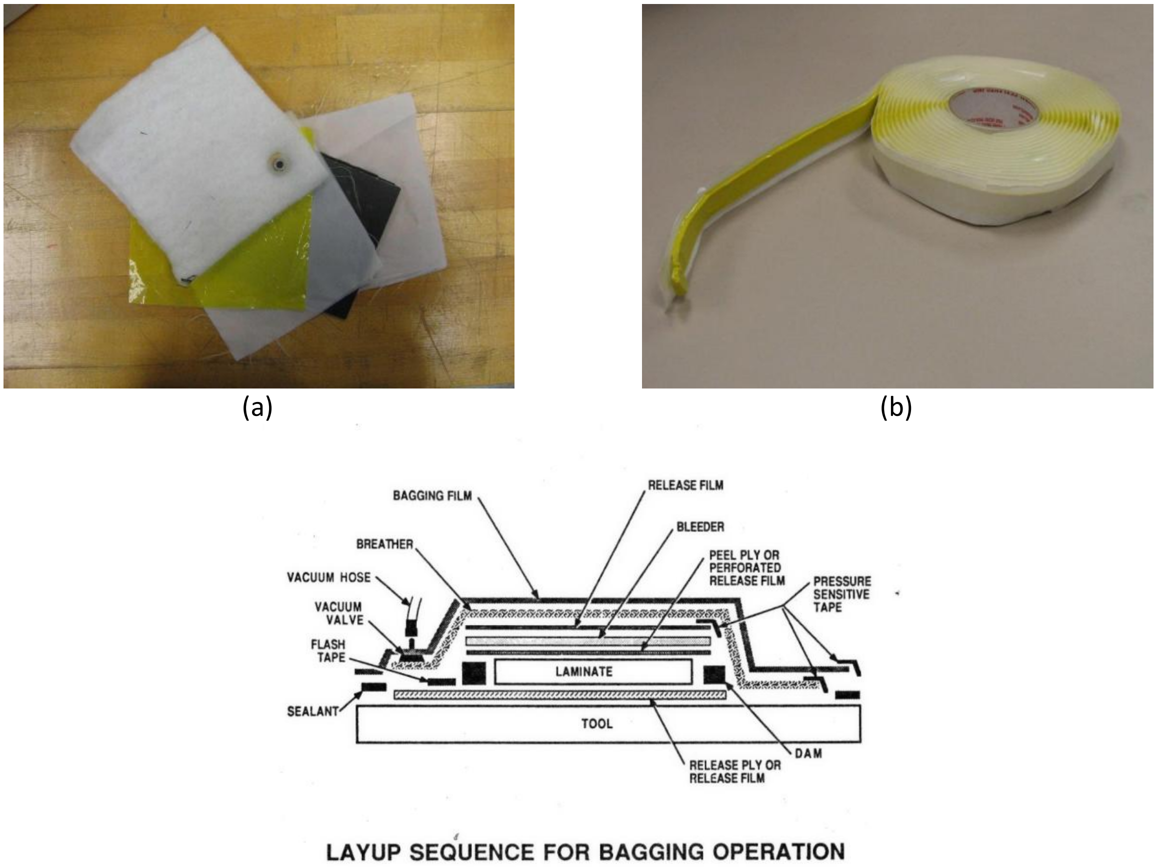
Figure 2. (a) Nonporous peel ply (b) woven fiberglass ply

An epoxy resin (Figure 3) was formed by mixing West System® Epoxy Resin 105 with Slow Hardener 206. The resin was evenly spread across the peel ply using a plastic applicator. After the first coat of resin had been applied, the composite was constructed by alternately stacking layers of woven fiber cloth (Figure 3) while applying coats of resin.

(a) (b)

Figure 3. (a) woven fiberglass cloth stacked to illustrated layup (b) epoxy

Following lay-up, a porous peel ply was placed on top of the stack, and a layer of breather material (Figure 4) was placed on top of the peel ply. The purpose of the breather material was to allow ventilation of any gases released during consolidation while simultaneously absorbing any resin that passed through the peel ply.



(a) (b)

(c)

Figure 4. (a) Hand layup materials; breather material shown on top (b) sealing tape (c) layup stacking sequence

A vacuum tube was placed adjacent to the stack, and the entire assembly was covered with a vacuum bag. The edges of the bag were sealed to the surface of the workstation with sealant tape (Figure 4), which provided a pressure-tight seal around the periphery of the assembly. To consolidate the composite, a vacuum was drawn for roughly twenty-four hours. After twenty-four hours, the vacuum bag, breather material, and peel plies were removed.

The material used and the layup for each composite panel is summarized in Table 1 and their material properties are summarized in Table 2.

Table 1. Composite material lay-ups

Panel No.	Material	Layup	Comments	# Plies
1	CF unidirectional	$[0/45/90/-45]_5$	cream	8
2	FG cloth	$[(0/90)_2/\underline{0}]_S$	much epoxy	9
3	CF cloth old	$[0]_7$	wavy threads	7
4	CF cloth	$[0]_8$	-	8
5	FG cloth	$[(0/45)_2/0/\underline{45}]_S$	-	11
6	CF unidirectional	$[(0/90)_2/\underline{0}]_S$	-	9
7	CF unidirectional	$[0]_9$	wavy tow	9
8	FG cloth	$[(0/45)_2/\underline{0}]_S$	-	9
9	CF cloth	$[(0/45)_2/\underline{0}]_S$	count plies b4 cutting	9
10	FG cloth	$[0]_{10}$	-	10
11	CF unidirectional/ FG cloth	$[0/90/FG/\underline{FG}]_S$	bubbles	4+3
12	FG chop	$[\,]_{13}$	Cream	13

Table 2. Physically measured material properties

Panel No.	Ply (g/mm^2)	Fiber (g/mm^2)	Fiber Density	Panel (g/mm^2)	Panel (g/mm^3)	Mass fraction	Thickness (mm)	Volume Fraction
1	1.61E-4	1.29E-3	1.8E-3	3.19E-3	1.36E-3	0.40	2.35	0.31
2	2.52E-4	2.27E-3	2.6E-3	3.08E-3	1.56E-3	0.74	1.97	0.44
3	2.01E-4	1.41E-3	1.8E-3	1.92E-3	1.36E-3	0.74	1.41	0.57
4	2.01E-4	1.61E-3	1.8E-3	2.37E-3	1.32E-3	0.68	1.80	0.51
5	2.52E-4	2.77E-3	2.6E-3	3.62E-3	1.55E-3	0.77	2.33	0.46
6	1.61E-4	1.45E-3	1.8E-3	4.63E-3	1.38E-3	0.31	3.36	0.25
7	1.61E-4	1.45E-3	1.8E-3	3.50E-3	1.39E-3	0.41	2.51	0.33
8	2.52E-4	2.27E-3	2.6E-3	3.11E-3	1.60E-3	0.73	1.95	0.45
9	2.01E-4	1.81E-3	1.8E-3	2.57E-3	1.33E-3	0.70	1.93	0.54
10	2.52E-4	2.52E-3	2.6E-3	2.40E-3	1.67E-3	1.05	1.44	0.67
11	2.52E-4	1.40E-3	1.8E-3	3.32E-3	1.38E-3	0.42	2.40	0.33
12	2.46E-5	3.19E-4	2.6E-3	3.51E-3	1.14E-3	0.09	3.08	0.04

3 Analysis Methodology

Classical Lamination Theory (CLT) was used to find the effective properties if a composite panel. CLT requires the sequence of laminate lay-up, volume proportions of the matrix and fibers, and material properties of both the matrix and fibers.

By using CLT to predict material properties of the panel some assumption had to be made. The first assumption is that the panel is thin, meaning the thickness is one-tenth the length and width. The second assumption is that the out-of-plane shear strains are zero, following the Kirchhoff Hypothesis. This hypothesis states that a line straight and perpendicular to the mid-plane stays straight and perpendicular after deformation. The third assumption is that all plies are perfectly bonded. This means that there are no voids or gaps between plies and no lamina can slip relative to another lamina.

3.1 Sample Calculations

After the panels were created the volume fraction was needed in order to find the effective properties of the panel from the fiber and matrix properties using rule of mixtures. First the volume of the fibers and volume of the matrix were calculated using Equation 1 and 2, respectively.

$$vol_f = \frac{m \times m_f}{\rho_f}$$ **Equation 1**

$$vol_m = \frac{m \times m_m}{\rho_m}$$ **Equation 2**

where m, m_f, $m_{m,}$ ρ_m, and ρ_f are the mass of the panel, mass fraction of the fibers, mass fraction of the matrix, density of the matrix, and the density of the fibers, respectively. The volume fraction, V_f, was then found for each panel using Equation 3 below.

$$V_f = \frac{vol_f}{vol_f + vol_m}.$$ **Equation 3**

Comparing typical carbon fiber and S-glass properties with our experimental results Young's Modulus and Poisson's ratio was found for both fibers. The matrix was made of West System 105/205 epoxy and properties for this system were available online. Table 3 summarizes these properties, all of the fibers of like kind were assumed to have the same properties.

Table 3. Assumed material properties used for CLT calculations.

	Material Properties		
	Carbon Fiber	**S-Glass**	**Epoxy Resin**
Young's Modulus (GPa)	347	58	3.17
Poisson's Ratio, ν_{12}	0.24	0.24	0.54

First the shear modulus for the fiber, G_f, and the matrix, G_m, were found using Equations 4 and 5.

$$G_f = \frac{E_f}{2(1+\nu_f)}$$ **Equation 4**

$$G_m = \frac{E_m}{2(1+\nu_m)}$$ **Equation 5**

Using these properties along with the properties tabulated in Table 3.1, rule of mixtures was used to find the effective properties of a unidirectional lamina. These equations are shown below, Equations 6 through 9.

$$E_{11} = E_m + V_f(E_f - E_m)$$ **Equation 6**

$$E_{22} = \frac{E_f E_m}{E_f - V_f (E_f - E_m)}$$ **Equation 7**

$$\nu_{12} = \nu_m - V_f (\nu_m - \nu_f)$$ **Equation 8**

$$G_{12} = \frac{G_f G_m}{G_f - V_f (G_f - V_m)}$$ **Equation 9**

E_{11}, E_{22}, ν_{12}, and G_{12} are the effective Young's Modulus along the fiber, transverse to the fiber, Poisson's Ratio, and shear modulus, respectively.

These effective lamina properties are used to find both the effective properties of a panel with multidirectional plies and fracture analysis.

Individual lamina properties are related to the effective laminate properties via the composite **ABD** matrix, which can be thought of as the *stiffness coefficient* in "Hooke's Law." The **ABD** matrix is of the form:

$$\boldsymbol{ABD} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix}$$ **Equation 10**

Where,
A is a 3x3 matrix whose terms are known as *extensional stiffnesses*, because they "relate in-plane stress resultants to in-plane mid-plane strains" [1].
B is a 3x3 matrix whose terms are known as *coupling stiffnesses*, because they "relate in-plane stress resultants to mid-plane curvatures" [1].
D is a 3x3 matrix whose terms are known as *bending stiffnesses*, because they "relate moment resultants to mid-plane curvatures" [1].

Once the effective lamina properties are known "CompositeABD", a Mathematica code, which is based on CLT can be used to find the effective laminate properties of multidirectional panels made of unidirectional lamina. When using a fiber cloth the 0 and 90° fibers need to accounted for by placing 'extra' lamina of the 90° orientation. For the analysis' of different orientations of the panel an extra line was added into the code to rotate directional lay-up. An example of this code is shown in Equation 11, for a panel orientation of 30° with 4 lamina.

PlyOrient = PlyOrient + {{30,30,30,30}} Degree **Equation 11**

For a more detailed sample calculation of CompositeABD refer to Appendix A.

To find the first ply failure of a panel 'LAMFAIL' can be used with the effective lamina properties and the failure stresses for the composite. These failure stresses were found in Prof. Tuttle's text and summarized in Table 4. Failure Stresses for Carbon Fiber and Fiber Glass. For a more detailed sample calculation of LAMFAIL the output of one of the calculations is in Appendix B.

Table 4. Failure Stresses for Carbon Fiber and Fiber Glass

	Carbon Fiber (Mpa)	Fiber Glass (Mpa)
σ11,ft	1500	1050
σ11,fc	1200	690
σ22,yt	50	45
σ22,yc	100	120
σ22,ft	70	55
σ22,fc	130	140
τ12,y	75	40
τ12,f	130	70

4 Experimental Methodology

Tensile testing was performed with an Instron tensile testing machine (Figure 5). The specimens were labeled, and their lengths, widths, and thicknesses were measured with calipers. They were then loaded into the grips of the Instron and positioned so that there was no torque. Displacement was measured with a gage length axial extensometer. The extensometer was attached at the start of the test, ensuring that the knife-edges bit into the specimen.

(a)

(b)

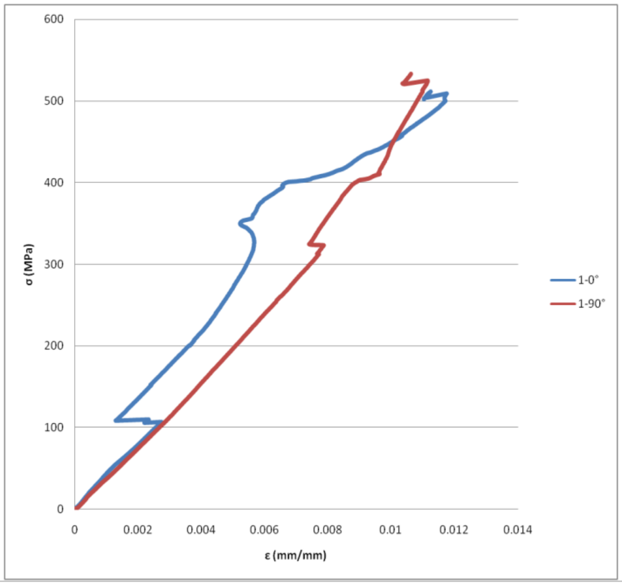
(c)

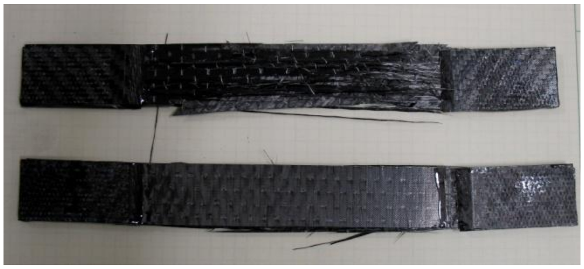
Figure 5. (a) Instron tensile test machine (b) tensile specimen with attached extensometer (c) tensile test specimens labeled and prepared for tensile test

The test was run under a continuous crosshead displacement control. A computer program collected the load and displacement data. The initial extension data was captured by the extensometer until strain was achieved. At this point, the extensometer was removed, and the crosshead displacement was recorded for the remainder of the test. Testing was complete when 50% of the maximum load had been reduced. The load and displacement data gathered by the Instron machine and measurements of the individual test specimens was reduced into the stresses and strains for each test strip. The slope of the linear region of the stress-strain curve was estimated to determine the specimen's modulus of elasticity.

5 Results and Discussion

Panel 1

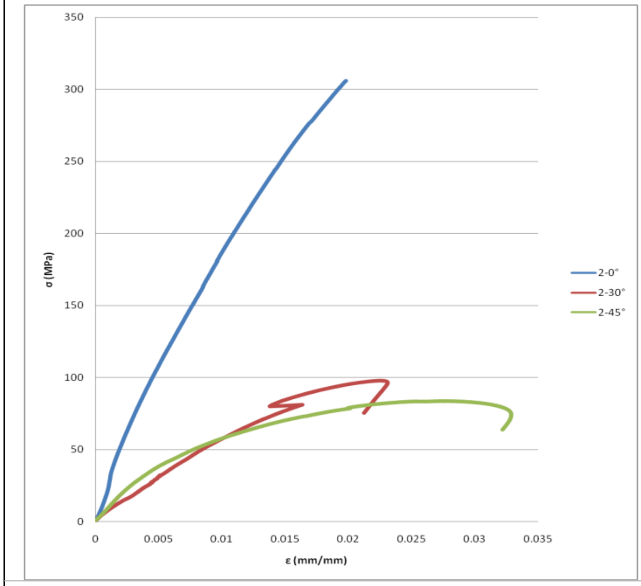


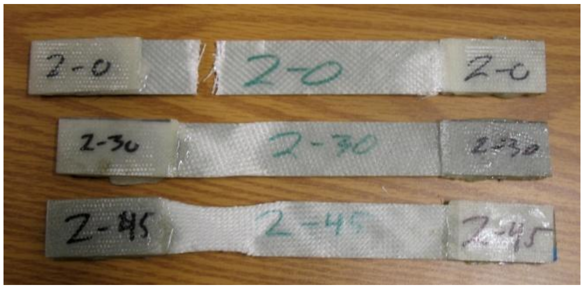


The two specimens from group 1 had similar failure modes. This is not surprising since their cut orientations were simply perpendicular to one another, and therefore, virtually identical. Inspection of the specimens reveals what appears to be a combination of delamination and fiber failure. Upon further examination it can be seen that the failure modes are identical, but the plies that fail are opposite. In the

1-0 specimen the 90° ply held while the others failure, in the 1-90 specimen the same is true, but now the first ply is at 90° instead of the middle plies.

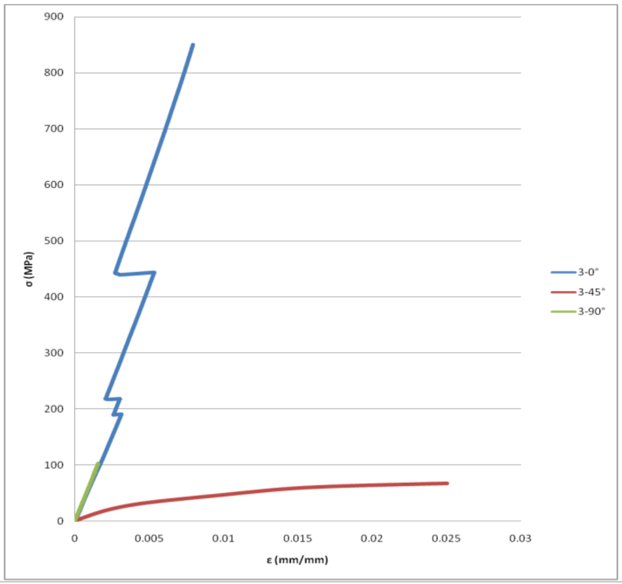
Panel 2





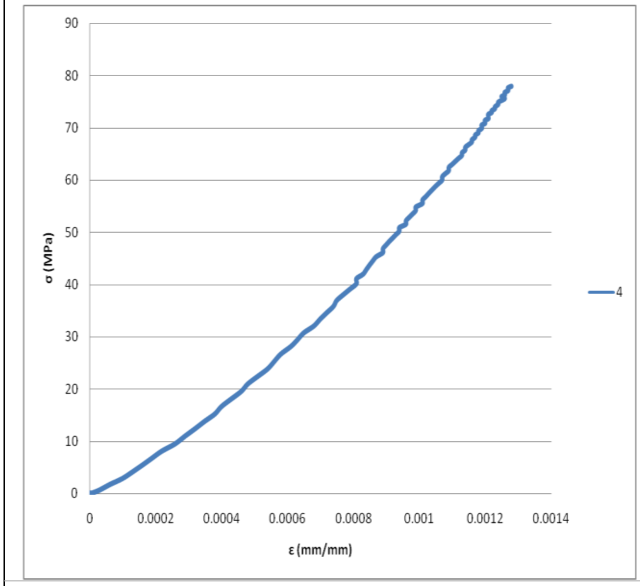
The specimens from group 2 exhibited different failure modes depending on the orientation at which they were cut. In other words, the failure mode depended on whether the direction of the applied load was off-axis from the direction of the fibers. Therefore, the 0° specimen experienced fiber failure since the load was being applied in the fiber direction. In contrast, the 30° and 45° specimens failed by yielding, since the applied load was off-axis from the direction of the fibers. This was evidenced by a shear band, necking, and extensive plastic deformation along the length of the specimens, especially in the 2-45° test specimen.

Panel 3



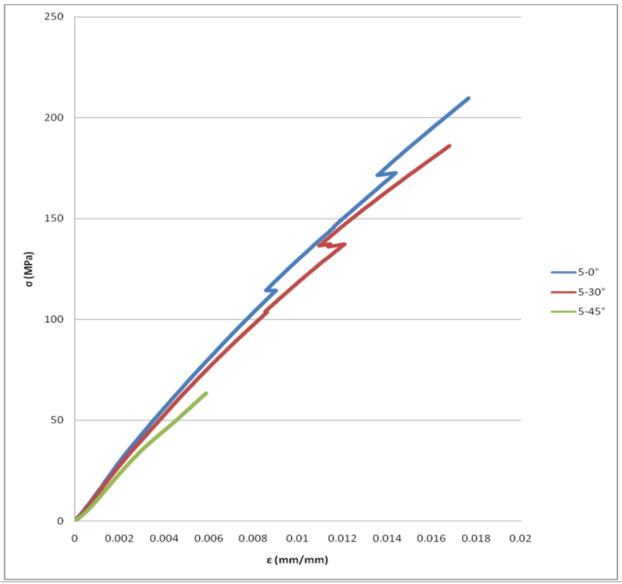
The lay-up of the group 3 panel was $[0]_7$. The specimens from group 3, like those of group 2, exhibited different failure modes depending on the orientation at which they were cut. Therefore, the 0° and 90° specimens experienced fiber failure since the load was being applied in the fiber direction. In contrast, the 45° specimen failed by yielding, since the applied load was off-axis from the direction of the fibers. This was evidenced by a shear band, necking, and extensive plastic deformation near one of the tabs of the specimen.

Panel 4



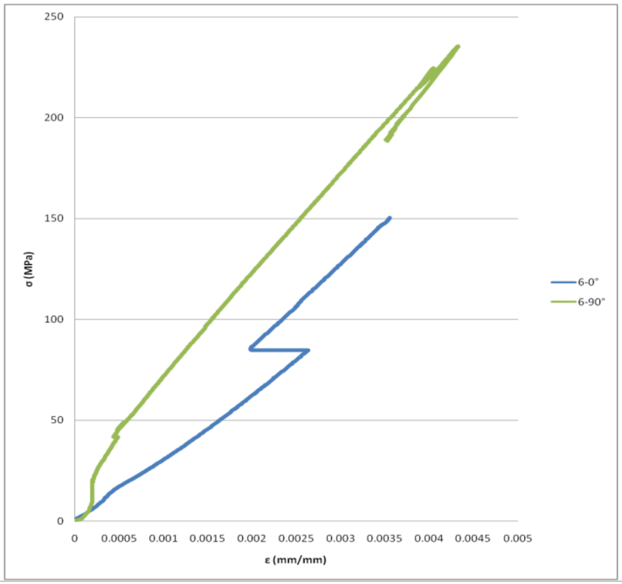
The stress-strain plot of panel 4 demonstrates what is already known by inspection of the test specimen: the grips slipped, thereby rendering the test invalid.

Panel 5



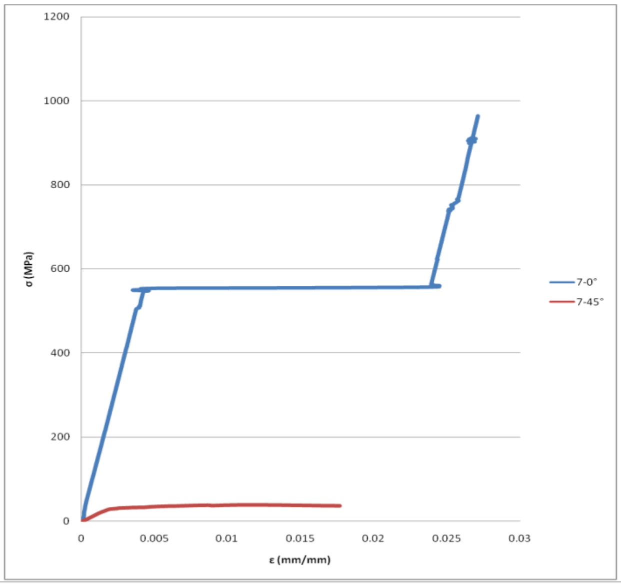
The group 5 panel had a $[(0/45)_2/0/\underline{45}]_S$ lay-up. The 45° specimen exhibited a combination of fiber and matrix failure. There was localized damage at the site of fracture and visible fiber pullouts on all specimens.

Panel 6



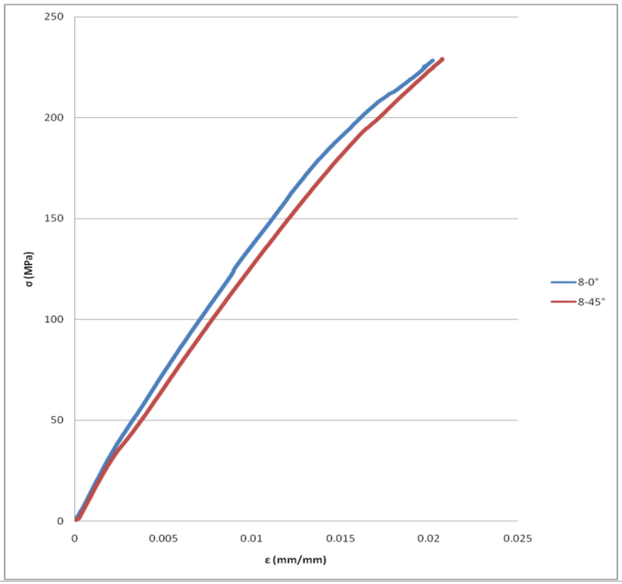
Both specimens from panel 6 experienced tab failure. No failure or deformation observed.

Panel 7



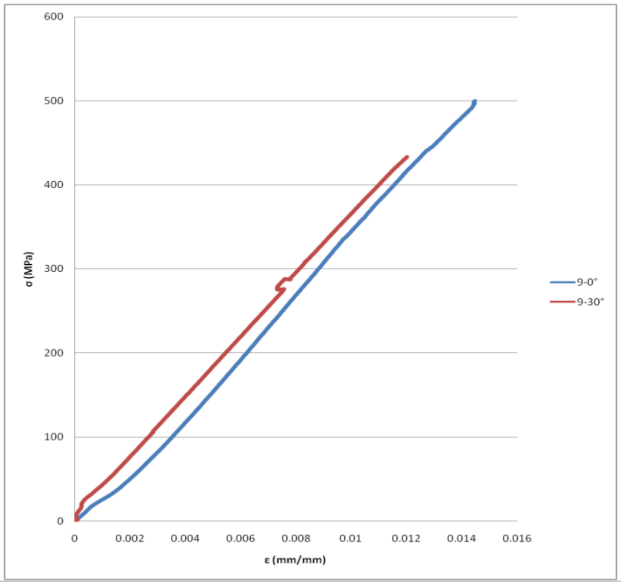
Group 7 was composed of CF unidirectional $[0]_9$ and was very strong in the 7-0° specimen, since the fibers were aligned with the tensile stress. Failure was not observed in either specimen, which is especially unfortunate for the 45° specimen, which likely have developed interesting deformation.

Panel 8



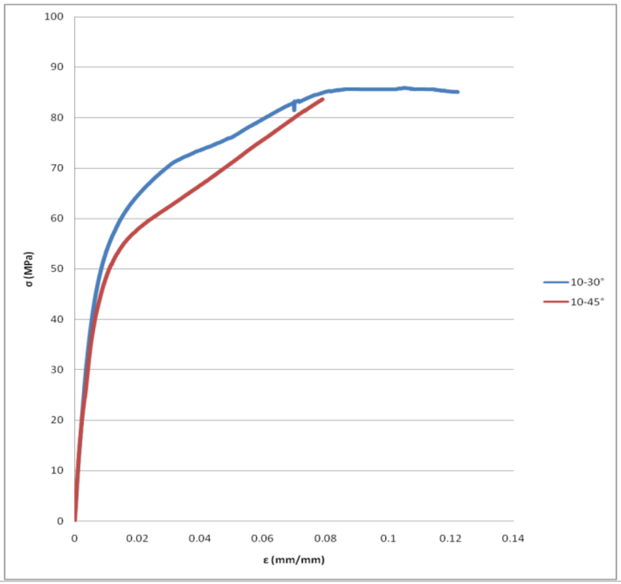
Panel 8 was composed of fiberglass cloth in a $[(0/45)_2/\underline{0}]_S$ layup orientation. Failure in the 0° specimen was initiated at the tab with massive delamination occurring in an angled band. On the 45° specimen, there was localized damage at the site of fracture and visible fiber pullouts.

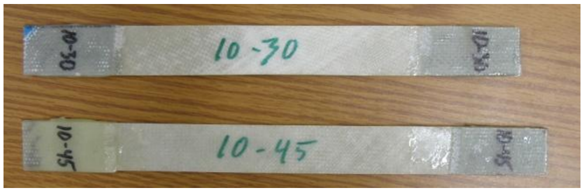
Panel 9



The lay-up of the group 9 panel was $[(0/45)_2/\underline{0}]_S$. The specimens from group 9, unlike those of group 3, did not exhibit noticeably different failure modes depending on the orientation at which they were cut. The 0° specimen experienced fiber failure since the load was being applied in the fiber direction. Although the 30° specimen did not have fibers aligned with the tensile axis, it too experienced fiber failure similar to the other, probably because the orientation of the fibers created properties that were nearly identical to 9-0°.

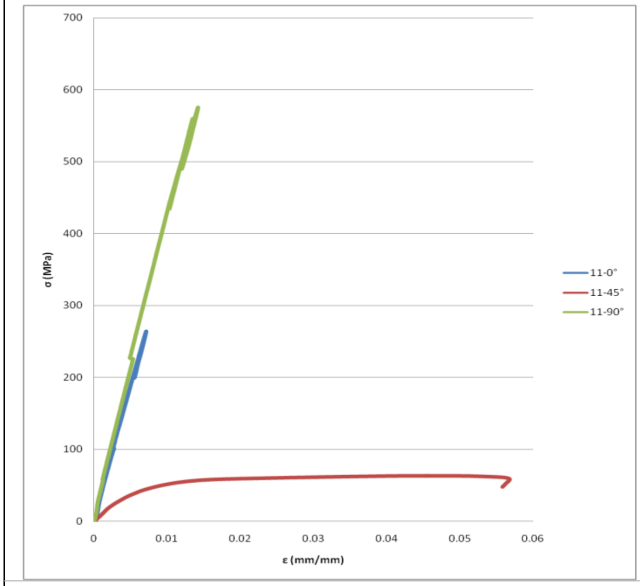
Panel 10





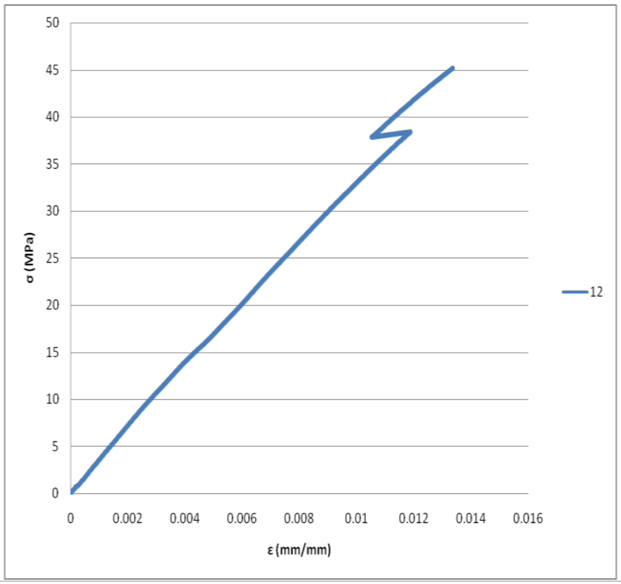
The group 10 panel had a $[0]_{10}$ lay-up. Since the panels were constructed with woven fiberglass cloth, each ply had fibers running in both the 0° and 90° directions. It was thus expected that the 30° and 45° specimens would exhibit similar failure models, namely, delamination. This was evidenced by extensive matrix failure, along the entire length of the 10-30° specimen, whereas the failure of 10-45° was more localized.

Panel 11



Panel 11 possessed a very unique layup. Since the layup was CF unidirectional/FG cloth $[0/90/FG/\underline{FG}]_S$, and because the fibers were aligned with the axis of applied stress, the 0° and 90° specimens were both very strong, and performed similarly. Furthermore, probably as a result of the high strength, both experienced tab failure. The 45° specimen, on the other hand, began yielding almost at the outset of applied tension. This was due to the obvious off-axis fiber orientation. The mode of failure, as illustrated above, is demonstrated in the 45° shearing appearance.

Panel 12



Since panel 12 is composed of chopped fiberglass, the specimen orientation would have no influence on strength properties. This panel withstood relatively little tensile force, and exhibited a low elastic modulus and it experienced fiber failure (in two places, in fact) perpendicular to the tensile axis. The failure in two locations is caused by a shockwave initiated when the first fracture occurs, causing a second on the other side of a specimen.

6 Summary

As mentioned throughout this paper there are many difficulties associated with testing composite panels. In order to predict the material properties of a panel the fiber and matrix properties must be known and unless a vendor states those properties clearly it can be hard to know what they are. Throughout the testing of these specimens it has been a guess and check scenario to find the fiber properties. We began by finding average properties for both the carbon fibers and glass fibers, then went back and checked how our predictions matched up with our results. From the testing results we were able to better identify what our fiber properties were. Table 5 summarizes our testing results along with our predictions.

The fiber properties were found by comparing predictions with results for the first two panels. Once satisfactory properties were found for both carbon fiber and fiber glass, these properties were used for all other panels. The assumption that all of the fibers would have the same properties is some cause for error. By looking at the percent error between the predicted Elastic Modulus and the measured, the first two panels have acceptable errors, while a few of the other panels have much larger errors.

Some of the other causes for error are the different modes of failure that CLT does not account for. It is clear that many of the panels experience delamination, this can be easily seen in many of the fiber glass specimens by a lighter colored band across the surface. Once delamination begins to occur the load is no longer transmitted through the matrix, between fibers which used to be bonded together. This will cause early failure of fibers experiencing off-axis loading.

Another cause for error is tab failure, which many of the specimens had. Tab failure causes a lack in data collection. If the tabs fail very early within the testing the elastic modulus can be very difficult to find, and if one is found it is hard to know how accurate it is. Also, no matter when the tab failure occurs, there is a lack in specimen failure data.

Table 5 Summary of Calculated Values

Panel No.	Cut Orientation (degrees)	Effective Elastic Modulus, E_{xx} (GPa)		Percent Error	Failure Load (N)	Ply
		Analytical	Measured			
1	0	39.4	40.2	1.99%	1.26E+06	1
	90	39.4	40.2	1.99%	1.26E+06	3
2	0	17.8	17.8	0.00%	3.60E+05	8
	30	7.5	4.97	50.91%	3.20E+05	9
	45	6.3	5.00	26.00%	2.76E+05	9
3	0	102.3	60.9	67.98%	1.29E+06	1
	45	9.5	*5.88*	61.56%	3.82E+05	1
	90	102.3	*66.6*	53.60%	1.29E+06	1
4	0	91.9	*49.4*	86.03%	1.31E+07	1
5	0	15.2	12.4	22.58%	3.85E+05	9
	30	12.5	12.2	2.46%	3.23E+05	6
	45	11.4	10.9	4.59%	2.85E+05	6
6	0	51.7	*50.9*	1.57%	2.28E+06	1
	90	42.2	*31.3*	34.82%	1.85E+06	2
7	0	117	133.8	12.56%	3.97E+06	1
	45	4.8	*15.1*	68.21%	3.70E+06	1
8	0	14.3	13.3	7.52%	3.10E+05	7
	45	12.1	12.5	3.20%	2.63E+05	8
9	0	71	36.5	94.52%	1.21E+06	1
	30	64.3	33.1	94.26%	1.06E+06	4
10	30	11.6	6.77	71.34%	2.66E+05	6
	45	9.9	5.61	76.47%	2.02E+05	10
11	0	-	38.1	-	1.19E+06	4
	45	-	8.33	-	3.40E+05	5
	90	-	42.7	-	1.19E+06	4
12	n/a	-	3.29	-	-	-

Note: an underscored elastic modulus value indicates a test specimen which slipped during testing; results are likely invalid

When keeping in mind all of these errors it is very difficult to know when a failure occurred within the specimen, except for last ply failure. Typical a failure is shown by a jump in the stress-strain graph, but unless very close attention is taken to each test it is hard to know why each jump is there. The jump could also be caused by the grips slipping on the tabs, or a tab slipping and the catching again, or the beginning of delamination, or first ply failure. Many of our stress-strain graphs show 2 or 3 jumps, so it is very difficult to know which one of these is the first ply failure.

The program that was used to predict failure gives first ply failure only. These predictions are often helpful to know which ply will fail first, but the load is hard to compare to anything since it is difficult to find where the first ply failure was from the collected test data. The predictions all seem reasonable, but there are many assumptions that go along with them. This is often a problem in industry as well, since it is difficult to very accurately predict failure using CLT, or even to know what properties to use.

In conclusion, using CLT is a very useful tool to predict the effective properties of a panel and to predict the failure of a panel. It is very important to understand what assumptions are made and what associated errors will be involved with using CLT. Composite materials are very beneficial to many products and they become very difficult to simulate even for basic structures.

7 Appendix A – ABDComposite

Analysis of composite laminates

P. Labossiere 30/1/2007

```
 Off[General::spell1];
Off[General::spell];
Clear["`*"]
```

Define the material properties by Type = 1)Known 2) Rule of Mixtures
Units MPa, mm, N

```
 Type=2;
If[Type==1,
  {E11=170000;
   E22=10000;
   □12=0.3;
   G12=13000;
   □11=-.9 10^-3;
   □22=27 10^-3;
   □11=50 10^-3;
   □22=1200 10^-3},
  {Vf=.31;
   Ef=374 10^9;
   □f=.24;
   Em=3.17 10^9;
   □m=.45;
   Gf=Ef/(2 (1+□f));
   Gm=Em/(2 (1+□m));
   E11=Em+Vf (Ef-Em);
   E22=(Ef Em)/(Ef-Vf (Ef-Em));
   □12=□m-Vf (□m-□f);
   G12=(Gf Gm)/(Gf-Vf (Gf-Gm))}];
```

Define the panel layout

```
 NumPly=9;
PlyThickness={{.219,.219,.219,.219,.219,.219,.219,.219,.219}};
PlyOrient={{0,45,0,45,0,45,0,45,0}} Degree;
PlyOrient = PlyOrient + {{45,45,45,45,45,45,45,45,45}} Degree;
```

Define the loads[N's and M's] and temperature and moisture change

```
 Load=Transpose[{{1,0,0,0,0,0}}];
DeltaT=0.0;
DeltaM=0.0;
```

Start a few precalculations and set up locations of the interfaces

```
 Q11=E11^2/(E11-□12^2 E22);
Q22=(E11 E22)/(E11-□12^2 E22);
Q12=(□12  E22  E11)/(E11-□12^2 E22);
```

```
Q66=G12;
MatrixForm[Qmat={{Q11,Q12,0},{Q12,Q22,0},{0,0,Q66}}];
Smat=Inverse[Qmat];
TotThick=Sum[PlyThickness[[1,i]],{i,1,NumPly}];
z=Table[0,{i,NumPly+1}];
z[[1]]=-TotThick/2;
Do[
 z[[k+1]]=z[[k]]+PlyThickness[[1,k]];,{k,1,NumPly}]
z
 {-0.9855,-0.7665,-0.5475,-0.3285,-
0.1095,0.1095,0.3285,0.5475,0.7665,0.9855}
```

Determine the ABD Matrix

```
 Clear[];
Qbar=Table[0,{k,1,NumPly}];
Tmat={{Cos[]^2, Sin[]^2, 2 Cos[]  Sin[]},{Sin[]^2,Cos[]^2,-2
Cos[]  Sin[]},{-Cos[]  Sin[],Cos[]  Sin[],Cos[]^2-
Sin[]^2}};
Rmat={{1,0,0},{0,1,0},{0,0,2}};
Do[zplus=zminus+PlyThickness[[1,k]];
  =PlyOrient[[1,k]];
  Sbar=Simplify[Rmat .Inverse[Tmat].Inverse[Rmat].Smat.Tmat];
  Qbar[[k]]=Inverse[Sbar];
  ,{k,1,NumPly}];
Amat=Table[0.,{i,3},{j,3}];
Bmat=Table[0.,{i,3},{j,3}];
Dmat=Table[0.,{i,3},{j,3}];
Amat=Sum[Qbar[[k]] (z[[k+1]]-z[[k]]),{k,1,NumPly}]//Chop;
Bmat=0.5 Sum[Qbar[[k]] (z[[k+1]]^2-z[[k]]^2),{k,1,NumPly}]//Chop;
Dmat=0.333 Sum[Qbar[[k]] (z[[k+1]]^3-z[[k]]^3),{k,1,NumPly}]//Chop;
ABD=Table[0.,{i,1,6},{j,1,6}];
Do[ABD[[i,j]]=Amat[[i,j]];ABD[[i+3,j]]=Bmat[[i,j]];ABD[[i,j+3]]=
Bmat[[i,j]];ABD[[i+3,j+3]]=Dmat[[i,j]];,{i,1,3},{j,1,3}]
MatrixForm[ABD]
 ( {
  {4.05157×10^10, 3.45772×10^10, 3.12639×10^10, -0.0000104904, -
8.58307×10^-6, -7.62939×10^-6},
  {3.45772×10^10, 1.4056×10^11, 3.12639×10^10, -7.62939×10^-6, -
0.0000257492, -7.62939×10^-6},
  {3.12639×10^10, 3.12639×10^10, 3.41974×10^10, -8.58307×10^-6, -
8.58307×10^-6, -8.58307×10^-6},
  {-0.0000104904, -8.58307×10^-6, -7.62939×10^-6, 1.5112×10^10,
1.31686×10^10, 1.21084×10^10},
  {-7.62939×10^-6, -0.0000257492, -7.62939×10^-6, 1.31686×10^10,
3.94786×10^10, 1.21084×10^10},
  {-8.58307×10^-6, -8.58307×10^-6, -8.58307×10^-6, 1.21084×10^10,
1.21084×10^10, 1.30457×10^10}
```

```
 } )
```

Determine the Effective Laminate Properties

```
 MatrixForm[abd=Inverse[ABD]]
Exxeff=1/(TotThick abd[[1,1]])
Eyyeff=1/(TotThick abd[[2,2]])
□xyeff=-abd[[1,2]]/abd[[1,1]]
□yxeff=-abd[[2,1]]/abd[[2,2]]
□xxxy=abd[[1,3]]/abd[[1,1]]
□yyxy=abd[[2,3]]/abd[[2,2]]
Gxyeff=1/(TotThick abd[[3,3]])
```

```
  ( {
   {8.61131×10^-11, -4.61035×10^-12, -7.45114×10^-11, 6.55552×10^-26, -
4.57558×10^-27, -5.79567×10^-26},
   {-4.61035×10^-12, 9.17715×10^-12, -4.17505×10^-12, -6.50023×10^-27,
6.1744×10^-27, 2.26309×10^-28},
   {-7.45114×10^-11, -4.17505×10^-12, 1.01179×10^-10, -4.94992×10^-26, -
1.13×10^-27, 6.75419×10^-26},
   {6.56106×10^-26, -4.25655×10^-27, -5.85449×10^-26, 2.67265×10^-10, -
1.82663×10^-11, -2.31109×10^-10},
   {-7.52577×10^-27, 6.1746×10^-27, 1.05401×10^-27, -1.82663×10^-11,
3.6659×10^-11, -1.70712×10^-11},
   {-4.93118×10^-26, -1.52251×10^-27, 6.81585×10^-26, -2.31109×10^-10, -
1.70712×10^-11, 3.07002×10^-10}
 } )
 5.89175×10^9
 5.52848×10^10
 0.0535383
 0.502373
 -0.865274
 -0.45494
 5.01446×10^9
```

8 Appendix B - LAMFAIL

```
          ***PROGRAM LAMFAIL*****


        WRITTEN BY PROF. MARK E. TUTTLE

       MECHANICAL ENGINEERING DEPARTMENT

          UNIVERSITY OF WASHINGTON


  THIS PROGRAM PERFORMS A FIRST-PLY FAILURE ANALYSIS FOR

  A MULTI-PLY, MULTI-ANGLE LAMINATE, BASED ON CLASSICAL

   LAMINATION THEORY AND SPECIFIED FAILURE CRITERION


   THE   1 MATERIALS PROPERTIES INPUT:


 MATL   E11       E22      NU12     G12

  1  0.110E+12  0.458E+10  0.38  0.158E+10


 MATL  ALP11     ALP22     BETA11    BETA22   THICKNESS

  1  0.000E+00 0.000E+00 0.000E+00 0.000E+00  0.000294


 LAMINATE DESCRIPTION:

    LAMINATE IS SYMMETRIC

    TOTAL NUMBER OF PLIES =   8

    TOTAL LAMINATE THICKNESS =  0.002352

    PLY    MATL              FIBER

    NO.    NO.    THICKNESS     ANGLE
```

```
   1    1    0.000294      0.00

   2    1    0.000294     45.00

   3    1    0.000294     90.00

   4    1    0.000294    -45.00

   5    1    0.000294    -45.00

   6    1    0.000294     90.00

   7    1    0.000294     45.00

   8    1    0.000294      0.00
 LAMINATE LOADING:


  THE FOLLOWING UNIT LOADS WERE DEFINED:

   Nxx      Nyy      Nxy       Mxx      Myy      Mxy

0.100E+01  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00


STRESS-FREE TEMPERATURE =     0.0

SERVICE TEMPERATURE =     0.0

CHANGE IN TEMPERATURE =     0.0

CHANGE IN MOISTURE CONTENT =  0.0
 STIFFNESS MATRICES:


  THE [Q] MATRIX FOR MATERIAL NUMBER  1:


   0.110665E+12    0.175093E+10    0.000000E+00

   0.175093E+10    0.460770E+10    0.000000E+00

   0.000000E+00    0.000000E+00    0.158000E+10
```

```
THE [QBAR] MATRIX FOR PLY NUMBER  1:

 0.110665E+12   0.175093E+10   0.000000E+00
 0.175093E+10   0.460770E+10   0.000000E+00
 0.000000E+00   0.000000E+00   0.158000E+10

THE [QBAR] MATRIX FOR PLY NUMBER  2:

 0.312737E+11   0.281137E+11   0.265144E+11
 0.281137E+11   0.312737E+11   0.265144E+11
 0.265144E+11   0.265144E+11   0.279428E+11

THE [QBAR] MATRIX FOR PLY NUMBER  3:

 0.460770E+10   0.175093E+10   0.000000E+00
 0.175093E+10   0.110665E+12   0.000000E+00
 0.000000E+00   0.000000E+00   0.158000E+10

THE [QBAR] MATRIX FOR PLY NUMBER  4:

 0.312737E+11   0.281137E+11  -0.265144E+11
 0.281137E+11   0.312737E+11  -0.265144E+11
-0.265144E+11  -0.265144E+11   0.279428E+11
```

```
 THE [QBAR] MATRIX FOR PLY NUMBER  5:


  0.312737E+11   0.281137E+11  -0.265144E+11

  0.281137E+11   0.312737E+11  -0.265144E+11

 -0.265144E+11  -0.265144E+11   0.279428E+11


 THE [QBAR] MATRIX FOR PLY NUMBER  6:


  0.460770E+10   0.175093E+10   0.000000E+00

  0.175093E+10   0.110665E+12   0.000000E+00

  0.000000E+00   0.000000E+00   0.158000E+10


 THE [QBAR] MATRIX FOR PLY NUMBER  7:


  0.312737E+11   0.281137E+11   0.265144E+11

  0.281137E+11   0.312737E+11   0.265144E+11

  0.265144E+11   0.265144E+11   0.279428E+11


 THE [QBAR] MATRIX FOR PLY NUMBER  8:


  0.110665E+12   0.175093E+10   0.000000E+00

  0.175093E+10   0.460770E+10   0.000000E+00

  0.000000E+00   0.000000E+00   0.158000E+10


THE [ABD] MATRIX IS:
```

```
 0.10456E+09  0.35121E+08  0.15946E+01  0.11499E-01  0.21783E-02 -0.21381E-03

 0.35121E+08  0.10456E+09  0.15946E+01  0.21783E-02  0.54888E-02 -0.21381E-03

 0.15946E+01  0.15946E+01  0.34719E+08 -0.21381E-03 -0.21381E-03  0.19092E-02

 0.11499E-01  0.21783E-02 -0.21381E-03  0.80512E+02  0.10831E+02  0.80855E+01

 0.21783E-02  0.54888E-02 -0.21381E-03  0.10831E+02  0.26609E+02  0.80855E+01

-0.21381E-03 -0.21381E-03  0.19092E-02  0.80855E+01  0.80855E+01  0.10646E+02

  THE [abd] MATRIX IS:

 0.10780E-07 -0.36211E-08 -0.32881E-15 -0.15846E-11  0.13006E-12  0.12486E-11

-0.36211E-08  0.10780E-07 -0.32881E-15  0.39582E-12 -0.26531E-11  0.18582E-11

-0.32881E-15 -0.32881E-15  0.28803E-07  0.50403E-12  0.22260E-11 -0.72391E-11

-0.15846E-11  0.39582E-12  0.50403E-12  0.13643E-01 -0.31262E-02 -0.79879E-02

 0.13006E-12 -0.26531E-11  0.22260E-11 -0.31262E-02  0.49574E-01 -0.35278E-01

 0.12486E-11  0.18582E-11 -0.72391E-11 -0.79879E-02 -0.35278E-01  0.12680E+00

   THERMAL STRESS AND MOMENT RESULTANTS ARE:

     Nxx      Nyy      Nxy       Mxx       Myy       Mxy

 0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00

  MOISTURE STRESS AND MOMENT RESULTANTS ARE:

     Nxx      Nyy      Nxy       Mxx       Myy       Mxy

 0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00

  FIRST-PLY FAILURE IS PREDICTED TO OCCUR IF LOADS

   ARE INCREASED BY A FACTOR OF   0.126E+07

   THAT IS, IF LOADS ARE INCREASED TO THE FOLLOWING VALUES:

     Nxx      Nyy      Nxy       Mxx       Myy       Mxy

 0.126E+07  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00

  THE PLY PREDICTED TO FAIL FIRST IS PLY NUMBER  1
```

```
   CALCULATED MIDPLANE STRAINS AND CURVATURES
   AT THE FIRST-PLY FAILURE LOAD ARE:
     EPSxx     EPSyy     GAMxy      kxx       kyy       kxy
  0.013627  -0.004577   0.000000 -0.000002   0.000000   0.000002
------------------------------------------------------------
 LAMINATE PLY STRESSES IN THE x-y COORDINATE SYSTEM
 AT THE FIRST-PLY FAILURE LOAD ARE:
 PLY NO   Z-COORD      SIGxx        SIGyy        TAUxy
------------------------------------------------------------
      -0.11760E-02   0.15000E+10   0.27692E+07  -0.35892E+01
  1
      -0.88200E-03   0.15000E+10   0.27692E+07  -0.28561E+01
------------------------------------------------------------
      -0.88200E-03   0.29748E+09   0.23995E+09   0.23994E+09
  2
      -0.58800E-03   0.29748E+09   0.23995E+09   0.23994E+09
------------------------------------------------------------
      -0.58800E-03   0.54774E+08  -0.48268E+09  -0.21229E+01
  3
      -0.29400E-03   0.54774E+08  -0.48268E+09  -0.13898E+01
------------------------------------------------------------
      -0.29400E-03   0.29748E+09   0.23995E+09  -0.23994E+09
  4
       0.58208E-10   0.29748E+09   0.23995E+09  -0.23994E+09
------------------------------------------------------------
```

```
      0.58208E-10   0.29748E+09   0.23995E+09  -0.23994E+09
 5
      0.29400E-03   0.29748E+09   0.23995E+09  -0.23994E+09
-------------------------------------------------------------
      0.29400E-03   0.54774E+08  -0.48268E+09   0.76428E-01
 6
      0.58800E-03   0.54774E+08  -0.48268E+09   0.80955E+00
-------------------------------------------------------------
      0.58800E-03   0.29748E+09   0.23995E+09   0.23994E+09
 7
      0.88200E-03   0.29748E+09   0.23995E+09   0.23994E+09
-------------------------------------------------------------
      0.88200E-03   0.15000E+10   0.27692E+07   0.15427E+01
 8
      0.11760E-02   0.15000E+10   0.27692E+07   0.22758E+01
-------------------------------------------------------------
 LAMINATE PLY STRESSES IN THE 1-2 COORDINATE SYSTEM
 AT THE FIRST-PLY FAILURE LOAD ARE:

 PLY NO   Z-COORD       SIG11        SIG22       TAU12
-------------------------------------------------------------
     -0.11760E-02   0.15000E+10   0.27692E+07  -0.35892E+01
 1
     -0.88200E-03   0.15000E+10   0.27692E+07  -0.28561E+01
-------------------------------------------------------------
```

```
    -0.88200E-03  0.50866E+09  0.28772E+08 -0.28762E+08

 2

    -0.58800E-03  0.50866E+09  0.28771E+08 -0.28762E+08

------------------------------------------------------------

    -0.58800E-03 -0.48268E+09  0.54774E+08  0.25616E+02

 3

    -0.29400E-03 -0.48268E+09  0.54774E+08  0.24883E+02

------------------------------------------------------------

    -0.29400E-03  0.50866E+09  0.28771E+08  0.28762E+08

 4

     0.58208E-10  0.50866E+09  0.28772E+08  0.28762E+08

------------------------------------------------------------

     0.58208E-10  0.50866E+09  0.28772E+08  0.28762E+08

 5

     0.29400E-03  0.50866E+09  0.28772E+08  0.28762E+08

------------------------------------------------------------

     0.29400E-03 -0.48268E+09  0.54774E+08  0.23416E+02

 6

     0.58800E-03 -0.48268E+09  0.54774E+08  0.22683E+02

------------------------------------------------------------

     0.58800E-03  0.50866E+09  0.28772E+08 -0.28762E+08

 7

     0.88200E-03  0.50866E+09  0.28772E+08 -0.28762E+08

------------------------------------------------------------

     0.88200E-03  0.15000E+10  0.27692E+07  0.15427E+01
```

```
 8

      0.11760E-02   0.15000E+10   0.27692E+07   0.22758E+01

-------------------------------------------------------------
```

9 References

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3. Tuttle, Mark E. “Composite Materials Analysis Programs.” 5 March 2009. < http://depts.washington.edu/amtas/computer.html>